Gold and WTI Crude Basket Option as a Hedge Against Financial and Industrial Crisis

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Abstract: Multi-asset options have many advantages over individual derivative options such as allowing investors to hedge against multiple risks and reducing transaction costs. Based on the price of gold and crude oil from 1986 to 2021, Geometric Brownian Motion Model (GBM) is introduced to predict gold Future price paths and WTI crude price paths. This article assumes that commodity prices exhibit randomness that can be explained and modeled by the GBM model and use the GBM model to simulate the future price paths for gold and WTI crude oil. To account for the correlation, the GBM of the second asset is modified. Then the average payout is determined and used as the estimator for the call price of the European basket option. Finally, sensitivity analysis is done by examining the following variables: correlation, risk-free rate, and volatility. As correlation decreases from positive to negative, the price of the call basket option also decreases. If the relationship between Gold and WTI crude has truly become decoupled in recent years then a call basket option of Gold and WTI crude will be cheaper and accessible to investors.

1. Introduction

Options can provide leverage and improve efficiency in the use of capital. Providing leverage is an important reason why options are attractive to investors. For call options, investors only need to pay a small portion of the underlying asset to gain the same return from the price movement of the underlying asset. Furthermore, the risk is limited, since investors can choose whether or not to exercise the option. For a call option, the most that an investor can lose is the price of the call option. In addition, options can create portfolios that are profitable under any market condition, which provides a pricing advantage over a single purchase of a derivative product.

This article specifically will focus on the price advantage of a basket option of Gold and WTI Crude over a portfolio of plain vanilla options of Gold and WTI Crude. The monthly historical prices of gold are taken from World Gold Council and the monthly historical prices of WTI Crude are taken from Federal Reserve Economic Database [1-2]. Annual returns, volatilities, and correlation are derived and used as estimators for the variables in the GBM model. Simulation and sensitivity analysis are then conducted to complete the study.

Correlations are especially important in this study. Investors analyze correlation to determine the degree of diversification and risk optimization for a portfolio. Thus, the correlation between the respective pricing of gold and oil is especially important since it helps investors determine their risk exposures to specific risks.

The price of oil influences the costs of production and manufacturing across the United States [3]. This is because higher oil prices increase the cost of goods through transportation costs. Higher costs of good are then passed on to the customers through the higher price of goods which reduce economic

growth. Or more directly, the price of oil directly affects the price of goods made from petroleum products [4]. Hence call options on WTI crude can be used to hedge against manufacturing risks such as rising cost of production.

The price of gold on the other hand is an indicator of the true health of the US economy. When the price of gold is high, it indicates an unhealthy economy. Investors buy gold to protect against economic crisis or inflation [5].

Figure 1 Gold Price From 1986 to 2021.

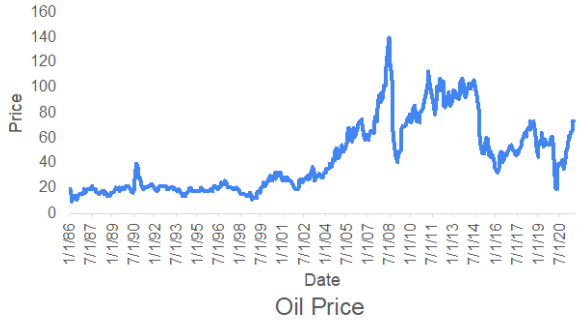

Figure 2 WTI Crude Price From 1986 to 2021.

Historically, there was a positive correlation between gold and crude oil. When oil prices rise based on above-ground barriers that limit production capacity, investors feel free to have investments such as gold and drives up the prices of gold. However, after 2014, oil prices stagnated, and gold prices continued to rise to lead to the collapse of the gold-oil price link [6].

The data in Figure 1 were used to analyze the correlations from 1986 to 2014, and from 2014 to 2021. The correlation is found to be positive until 2014, however, the correlation continues to weaken after 2014.

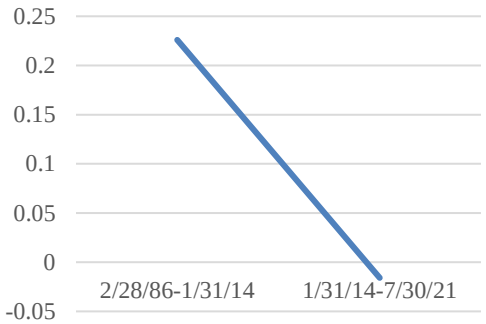

Figure 3 Gold and Crude Oil Correlation during 1986 to 2014 and 2014 to Present.

The primary objective of this paper is to identify the pricing advantages of a call basket option of Gold and WTI crude compare to plain vanilla call options on Gold and WTI crude. A secondary objective is to identify the parameters in which our model can reasonably predict and the price of a

call basket option of Gold and WTI crude. Lastly, highlight the significance of the change in the trend of correlation between Gold and Crude Oil and its impact on our model and pricing.

To accomplish these objectives, this paper is broken down into five parts GBM model, Results and Analysis, Sensitivity Analysis, Discussion, Conclusion. The first two sections will address the mathematical models for pricing and the pricing advantage identified. The third and fourth sections will address the process used to identify the parameter fit for the practical application of our mathematical model. The last section will identify the importance of the change in the trend of correlation between Gold and WTI crude in recent years and further research directions to improve our models and findings.

2. Geometric Brownian motion (GBM) model

In this section, to predict gold future price paths and WTI crude price paths, we introduce the Geometric Brownian Motion model (GBM). GBM model is a stochastic process that assumes independent stock returns are normally distributed [7]. The GBM model is known for its application in stock price modeling and options modeling. Popular models for option pricing such as the Black Scholes Merton formula utilizes GBM as assumptions. Many studies have shown that simulated stock prices align with actual stock prices, others also show that commodity prices also exhibit randomness which can be explained by the GBM model such as oil and gold [8].

For this study, we will assume that commodity prices exhibit randomness that can be explained and modeled by the GBM model and use the GBM model to future price paths for gold and WTI crude oil. Then we will utilize these future prices to evaluate the playout at maturity of a European basket option of the gold and WTI crude.

2.1. Theorem I

A stochastic process S(t) follows a geometric Brownian motion if the asset, specifically commodities gold and WTI crude satisfies the following stochastic differential equation [9].

$$dS_t = \mu S_t dt + \sigma S_t dW_t \tag{1}$$

The instantaneous change in S(t) spot price of an asset is comprised of the following variables μ, the risk-free interest rate, σ the volatility, and W_t the random Brownian motion. Using the following theorem we can derive the future price of an asset [10].

$$S_t = S_0 * Exp(\mu t - \frac{\sigma^2}{2} t + \sigma W_t) \tag{2}$$

S_t, the future price of an asset can be modeled by S_0 the current price of an asset, and the exponential distribution with the following variables the risk-free rate, volatility, time, and the random Brownian motion. This is the underlying model we will be using to simulate future prices of gold and WTI crude.

2.1. Theorem II

W_t the random Brownian motion follows a standard normal distribution with a mean of zero and standard deviation of one [11].

$$W_t \sim N(0,1) \tag{3}$$

By generating a thousand random standard normal numbers for W_t the random Brownian motion is generated and commodity future price paths are derived. However, to account for the correlation between gold and WTI crude, the second asset's W_t the random Brownian motion is modified [12].

$$W_2(t) = \rho W_1(t) + \sqrt{1-\rho^2} W_3(t) \tag{4}$$

$W_2(t)$ the random Brownian motion for the second asset WTI crude is comprised of ρ the correlation between gold and WTI crude, $W_1(t)$ the random Brownian motion generated for gold, $W_3(t)$ the standard normal number for the Brownian motion generated for an independent asset.

3. Results and Analysis

From the simulation results of future gold and WTI crude prices, the payouts at maturity are calculated and the mean payout is determined and used as an estimator for the European basket option price. The estimated price appears to be cheaper compare to hedging against each asset separately with plain vanilla options.

Intuitively this is consistent with the principle of diversification. Option prices are dependent on the volatility of the underlying asset. Higher volatility will result in a higher probability of a higher payout which in term will lead to a higher cost. However, a basket option's volatility is dependent on the volatility of the whole portfolio. Since gold and WTI crude demonstrate an imperfect weak positive correlation, there is an effective diversification of assets in the portfolio, which reduces the volatility exposure at a given return. The reduced volatility will hence reduce the payout at maturity providing investors a cheaper option to insure against their portfolios instead of insuring against gold and WTI crude separately.

Furthermore, the transaction costs required to ensure against each asset are minimalized. Instead, of having to purchase each gold and WTI crude option separately paying for at least two contract's transaction costs, investors would only need to purchase one contract to hedge against both.

4. Sensitivity analysis

4.1. Price Sensitivity to Correlation

First, the correlation is analyzed. Keeping other variables unchanged, we selected a range of correlation coefficients from negative one to positive one and derived the call price of the basket option for each new correlation coefficient. The results were plotted in Figure 4. When the correlation is negative, the call option is cheaper, and when it is positive, the call option is more expensive. In Figure 4, the price of the call option remains relatively stable between $90.27 to $96.44 when the correlation coefficient is between 0 to 0.4. However, when the correlation becomes negative from 0 to -0.4, the price of the call option fluctuates between $67.32 and $90.27. Given the same range in correlation when the correlation is negative the range in call prices is $16.78 greater than when the correlation is positive.

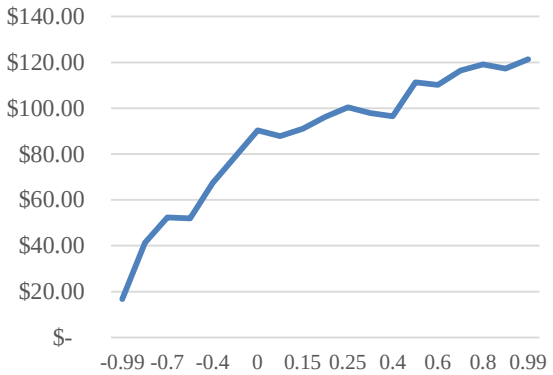

Figure 4 Call Price vs Correlation

Hence it is recommended that investors use this simulation method and pricing model only when investor assumes the correlation coefficient between gold and crude oil is between 0 to 0.4. When the correlation coefficient between gold and crude oil significantly deviates from this range, then the pricing of this model may be too significantly impacted by an error in the estimation of the correlation.

4.2. Price Sensitivity to Risk-Free Rate

For the simulation, 0.05% risk-free interest rate based on three months treasury yield was selected from the US Department of Treasury [13]. In the sensitivity analysis of the risk-free rate, the call option price was recalculated with 0.01% to 4% as the risk-free rate and the results are plotted below in Figure 5. However, because the short-term fluctuation of risk-free rate is very small, although some results

fluctuate in the line chart, the overall fluctuation range is not large. When the variation range is very small, the stock price can be approximately seen as changing parallel to the X-axis, so we believe that the risk-free rate has almost no influence on the results. Investors do not need to consider the effect of the risk-free interest rate on pricing when the time to maturity is short.

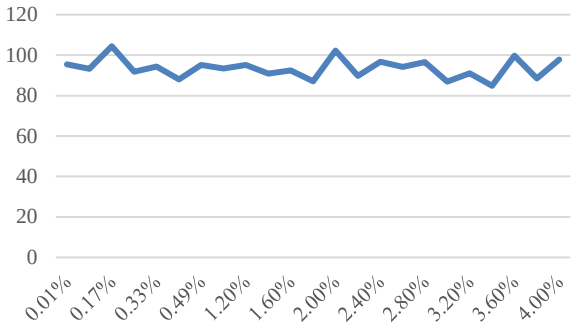

Figure 5 Call Price vs Risk-Free Rate

4.3. Price Sensitivity to Volatilities

Finally, sensitivity analysis concerning volatilities is carried out. The volatility of gold and crude oil used in the simulation is 0.1534 and 0.3713, respectively. Figure 6 is the 3D plot of call price for the volatilities of Gold and WTI Crude. The call price of the basket option is extremely sensitive to the change in volatility of either gold or WTI crude oil. The 3D plot shows that for the price of the actual call option to be within $5 of our estimated price of the call option, a careful combination of volatilities has to be maintained. However, our basket call option is more sensitive to gold volatility compare to WTI crude volatility. In Figure 6 when gold volatility exceeds 0.2 regardless of WTI crude's volatility, call option prices will always exceed $100. On the other hand, for WTI crude's volatility between 0 to 0.46 there always exists a combination of gold volatility that will result in a call price that's with $5 of our estimated price of the call option.

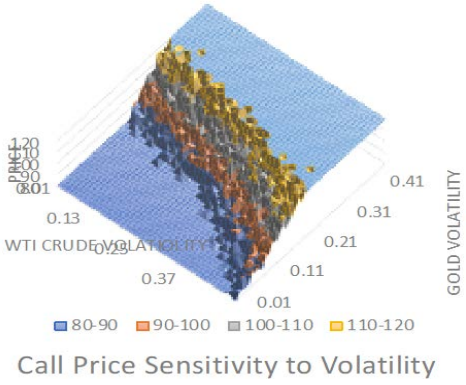

Figure 6 Call Price Sensitivity to Volatility

5. Discussion

For the most part, the dynamic correlation between gold and oil markets is positive. Because the price of oil increases the cost of commodities, the price of commodities increases, and the value of the dollar decreases. When the dollar falls, gold rises. Changes in gold and oil prices are complex and changeable, and investors should consider a variety of factors, such as correlation and volatility when making investment choices [14]. The price of gold is not only affected by the relationship between the supply and demand of goods but also by economic and political changes. As assets with strong financial and political attributes, their price trends are closely related to the global economic level. There are different national policies at different times and places. Investors should consider real-time political and economic factors and make selective investments comprehensively. It is also important

to choose different investment ratios, which in the case of gold and oil imply different risks. According to the results of the above-simulated price and combined with their conditions, investors can choose to buy options that meet their expectations.

6. Conclusions

Diversification occurs when the correlation between assets has zero or an imperfect correlation. Gold prices and WTI crude prices have historically exhibited a slight positive correlation of 0.1596 from January 31st, 1986 to July 30th, 2021. However, in recent years the correlation between Gold prices and WTI crude prices has changed.

A breakdown of the period shows that from January 31st, 2014 to present Gold prices and WTI crude prices exhibited a close to zero correlation of -0.0159. If the relationship between Gold prices and WTI crude prices has changed then can effective diversification of Gold and WTI crude occurs. In this study, we assumed that this diversification can occur and simulated, analyzed the practical implication of diversification on basket options of Gold and WTI crude.

Utilizing historical correlation, a call basket option of Gold and WTI crude resulted in cheaper pricing than compares to purchasing separate vanilla call options of Gold and WTI Crude. However, this difference in price increases, even more, when the correlation is closer to zero or negative. In the sensitivity analysis, it was shown that as correlation decreases from positive to negative so does the price of the call basket option. Hence, if the relationship between Gold and WTI crude has truly become decoupled and independent then the basket option of Gold and WTI crude can truly offer investors a bargain to hedge against financial and industrial risks.

The observations made in this paper rely on several assumptions on correlation and Geometric Brownian Motion that needs to be further researched for the conclusions made to be reliable. The change in the correlation between GOLD prices to WTI crude should be further examined since cycles of rising and decline of gold prices tends to occur over a decade or more. The short time frame from 2014 to 2021 may not have been sufficient to demonstrate a significant decoupling of WTI crude prices and Gold prices. Furthermore, the GBM model utilizes in this paper does not account for the time-dependent volatility and volatility clustering that occurs for Gold and WTI crude commodities. Further research should be done to incorporate volatility clustering into modeling Gold and WTI crude prices.

References

[1] World Gold Council. (2021, September 20). *Gold price historical data: Gold price history*. World Gold Council. Retrieved September 26, 2021, from https://www.gold.org/goldhub/data/gold-prices.

[2] U.S. Energy Information Administration, Crude Oil Prices: West Texas Intermediate (WTI) - Cushing, Oklahoma [DCOILWTICO], retrieved from FRED, Federal Reserve Bank of St. Louis; https://fred.stlouisfed.org/series/DCOILWTICO, September 25, 2021.

[3] Balke, N. S., & Brown, S. P. A. (2018, February 28). *Oil supply shocks and the U.S. economy: An estimated DSGE model*. Energy Policy.

[4] Federal Reserve Bank of San Francisco. (2007, November 1). *What are the possible causes and consequences of higher oil prices on the overall economy?* Federal Reserve Bank of San Francisco.

[5] Amadeo, K. (n.d.). *What makes gold prices go up and down*. The Balance. Retrieved September 21, 2021,

[6] Aguilera, R. F., & Radetzki, M. (2017, September 22). *The synchronized and exceptional price performance of oil and gold: Explanations and prospects*. Resources Policy.

[7] Ibrahim, S. N. I., Misiran, M., & Laham, M. F. (2021). Geometric fractional Brownian motion model for commodity market simulation. *Alexandria Engineering Journal*, *60*(1), 955-962.

[8] Ibrahim, S. N. I., Misiran, M., & Laham, M. F. (2021). Geometric fractional Brownian motion model for commodity market simulation. *Alexandria Engineering Journal*, *60*(1), 955-962.

[9] Deelstra, G. (2003). Pricing of arithmetic basket and Asian basket options by conditioning.

[10] Deelstra, G. (2003). Pricing of arithmetic basket and Asian basket options by conditioning.

[11] Iancu, A. K. (2004). *Numerical methods for pricing basket options*. The Ohio State University.

[12] Iancu, A. K. (2004). *Numerical methods for pricing basket options*. The Ohio State University.

[13] U.S. Department of the Treasury. (2021, September 24). Retrieved from https://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yield

[14] Li PP & DONG ZL (2020). *Time-varying network analysis of fluctuations between crude oil and Chinese and US gold prices in different perio*